Soliton Solutions of M–theory on an Orbifold

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Abstract

We explicitly construct soliton solutions in the low energy description of M–theory on S^1/Z_2. It is shown that the 11–dimensional membrane is a BPS solution of this theory if stretched between the Z_2 hyperplanes. A similar statement holds for the 11–dimensional 5–brane oriented parallel to the hyperplanes. The parallel membrane and the orthogonal 5–brane, though solutions, break all supersymmetries. Furthermore, we construct the analog of the gauge 5–brane with gauge instantons on the hyperplanes. This solution varies nontrivially along the orbifold direction due to the gauge anomalies located on the orbifold hyperplanes. Its zero mode part is identical to the weakly coupled 10–dimensional gauge 5–brane.

1   Introduction

One of the most interesting physical consequences of string duality is the description of
strongly coupled heterotic string theory as M–theory on S^1/Z_2 [1]. The low energy limit
of this theory has been constructed as 11–dimensional supergravity coupled to two 10–
dimensional E_8 super–Yang–Mills theories on the two orbifold fixed hyperplanes [2]. This
construction allows one to study some of the physics in the strongly coupled region of
the heterotic string, despite the fact that the fundamental underlying theory is still not
fully known. More precisely, the effective action of ref. [2] has been constructed as an
expansion in powers of the 11-dimensional Newton constant κ where terms up to the first
nontrivial order, that is $\kappa^{2/3}$ relative to 11–dimensional supergravity, have been taken into
account.

A wide class of vacuum solutions to the field-theoretical model of [1], which are relevant
for a dimensional reduction down to $D = 4$ theories with residual $N = 1$ supersymmetry,
has been constructed in ref. [3]. These solutions correspond to Calabi–Yau 3–folds times
S^1/Z_2 times 4–dimensional Minkowski space in the zeroth order in κ. Once terms of order
$\kappa^{2/3}$ are taken into account, the Calabi–Yau space gets deformed due to the nontrivial
structure of the 3–form Bianchi identity.

In recent years, soliton solutions have played a crucial role in the study of dualities
and nonperturbative effects in string and field theories. In particular, such solutions have
been constructed for weakly coupled heterotic string theory [4, 5, 6, 7]. If M–theory on
S^1/Z_2 indeed describes the strong coupling limit of heterotic string theory, one expects
to find the counterpart of those solutions as solitons of M–theory on S^1/Z_2. Therefore,
it is of interest to find the explicit soliton solutions of the field theoretical low energy
description of M-theory on S^1/Z_2 constructed in [1].

In this paper, we are going to construct several fundamental solutions for M-theory on
S^1/Z_2, namely explicit solitonic solutions that preserve a fraction of the supersymmetries
of the original theory. Furthermore, we will compute these solitonic solutions to order $\kappa^{2/3}$,
thus including the non-trivial effects of the gauge and gravitational anomalies. Specifically,
we will discuss the membrane, 5-brane and gauge 5-brane solitons of M-theory on S^1/Z_2.
Having these solutions in an explicit form helps in verifying duality relations to other
models. Finally, it is interesting to study the implications of the nontrivial boundary
conditions at the hyperplanes on the amount of supersymmetry supported by the solution.

2   General properties of the theory

We start with a brief overview of M–theory on S^1/Z_2 which describes the low energy
limit of the strongly coupled heterotic string theory [2]. The 11–dimensional coordinates
are denoted by $x^0, ..., x^9, x^{11}$. We take x^{11} as the orbifold direction and choose the range

$x^{11} \in [-\pi\rho, \pi\rho]$ with a periodic identification $x^{11} \sim x^{11} + 2\pi\rho$ of the endpoints. The Z_2 symmetry acts as $x^{11} \to -x^{11}$ and, therefore, gives rise to two 10–dimensional fixed hyperplanes at $x^{11} = 0$ and $x^{11} = \pi\rho$ respectively. The effective action for M–theory on S^1/Z_2 describes the coupling of two 10–dimensional E_8 super–Yang–Mills theories on these hyperplanes to 11–dimensional supergravity in the bulk. Let us denote 11–dimensional indices by $I, J, K, \dots = 0, \dots, 9, 11$ and 10–dimensional indices by $A, B, C, \dots = 0, \dots, 9$. Then the bosonic part of this action is specified by

$$S = \int d^{11}x \left(\mathcal{L}_{\text{SG}} + \delta(x^{11})\mathcal{L}_{\text{YM}} \right) , \tag{1}$$

with

$$\begin{aligned}
\mathcal{L}_{\text{SG}} &= \frac{1}{2\kappa^2}\sqrt{-g}\left[-R - \frac{1}{24}G_{IJKL}G^{IJKL} \right] \\
&\quad - \frac{1}{\kappa^2}\frac{\sqrt{2}}{3456}\epsilon^{I_1 \dots I_{11}} C_{I_1 I_2 I_3} G_{I_4 \dots I_7} G_{I_8 \dots I_{11}} + \cdots
\end{aligned} \tag{2}$$

and

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4\lambda^2}\sqrt{-\,^{10}g}\,\text{tr}(F_{AB}F^{AB}) + \cdots \tag{3}$$

where $\lambda^2 = 2\pi(4\pi\kappa^2)^{2/3}$. Here, the dots indicate the omitted fermionic terms which will be of no importance for the purpose of this paper. $\mathcal{L}_{\text{SG}}$ is the usual Lagrangian of 11–dimensional supergravity [8]. The "boundary" Lagrangian $\mathcal{L}_{\text{YM}}$ describes an E_8 super–Yang–Mills theory at $x^{11} = 0$ plus additional (fermionic) terms which result from the coupling to 11–dimensional supergravity. The metric $^{10}g_{AB}$ is the restriction of the 11–dimensional metric g_{MN} to the Z_2 hyperplane. For simplicity, we have concentrated on the hyperplane at $x^{11} = 0$. The contribution from the hyperplane at $x^{11} = \pi\rho$ adds in an obvious way to the action (1) as well as to the following formulae. The above action has to be supplemented with the nontrivial Bianchi identity

$$(dG)_{11ABCD} = -3\sqrt{2}\frac{\kappa^2}{\lambda^2}\delta(x^{11})\left(\text{tr}(F_{[AB}F_{CD]}) - \frac{1}{2}\text{tr}(R_{[AB}R_{CD]}) \right) , \tag{4}$$

for the 4–form field strength G_{IJKL}. It has been derived in ref. [2] from the requirements of anomaly cancellation and supersymmetry and it is designed to reproduce the analogous equation for the heterotic string upon reduction to 10 dimensions. In particular, the factor of $1/2$ in front of the $\text{tr}(R^2)$ serves to distribute the total gravitational contribution equally to the two hyperplanes. The Bianchi identity (4) can be solved in terms of the 3–form field C_{IJK} and the gravity and Yang–Mills Chern–Simons forms $\omega_{ABC}^{(L)}, \omega_{ABC}$ as follows

$$G_{ABCD} = \partial_A C_{BCD} \pm 23 \text{ perm.} \tag{5}$$

$$G_{11BCD} = (\partial_{11}C_{BCD} \pm 23 \text{ perm.}) + \frac{\kappa^2}{\sqrt{2}\lambda^2}\delta(x^{11})(\omega_{BCD} - \frac{1}{2}\omega_{BCD}^{(L)}) . \tag{6}$$

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Explicitly, the Yang–Mills Chern–Simons form is given in terms of the gauge field as

$$\omega_{ABC} = \text{tr}\left[A_A(\partial_B A_C - \partial_C A_B) + \frac{2}{3} A_A[A_B, A_C] + \text{ cyclic perm.}\right] . \tag{7}$$

A similar expression holds for $\omega_{ABC}^{(L)}$. Let us now collect the bosonic equations of motion to be derived from the action (1) which we are going to need for our discussion of soliton solutions. For the explicit examples, we will find that the gravitational contribution to the anomaly in eq. (4) vanishes. Consequently, we drop the resulting terms from the following equations of motion. The Einstein equation is given by

$$R_{MN} - \frac{1}{2}g_{MN}R + \frac{1}{6}\left(G_{MJKL}G_N{}^{JKL} - \frac{1}{8}g_{MN}G_{IJKL}G^{IJKL}\right) + \delta(x^{11})T_{MN} = 0 , \tag{8}$$

where the only nonvanishing components of the Yang–Mills stress energy tensor T_{MN} are

$$T_{AB} = \frac{\kappa^2}{\lambda^2}(g_{11,11})^{-\frac{1}{2}}\left[\text{tr}(F_{AC}F_B{}^C) - \frac{1}{4}g_{AB}\text{tr}(F_{CD}F^{CD})\right] . \tag{9}$$

The 3–form equations of motion read

$$\partial_M\left(\sqrt{-g}G^{MNPQ}\right) - \frac{\sqrt{2}}{1152}\epsilon^{NPQI_4...I_{11}}G_{I_4...I_7}G_{I_8...I_{11}}$$
$$+ \frac{\kappa^2}{\lambda^2}\frac{1}{432}\delta(x^{11})\epsilon^{11NPQA_5...A_{11}}\omega_{A_5A_6A_7}G_{A_8...A_{11}} = 0 . \tag{10}$$

Finally, for the gauge fields we have

$$D_A F^{iAB} - \frac{1}{\sqrt{2}}(g_{11,11})^{\frac{1}{2}}F^i_{DE}G^{11BDE} - \frac{1}{1152}\frac{1}{\sqrt{-^{10}g}}\epsilon^{BB_2...B_{10}}A^i_{B_2}G_{B_3...B_6}G_{B_7...B_{10}} = 0 , \tag{11}$$

with the space–time and gauge covariant derivative D_A. It is important to restrict the solutions of the above set of equations to those which respect the Z_2 orbifold symmetry. This Z_2 invariance implies for the fields

$$\begin{aligned}
g_{AB}(x^{11}) &= g_{AB}(-x^{11}) & G_{ABCD}(x^{11}) &= -G_{ABCD}(-x^{11}) \\
g_{A11}(x^{11}) &= -g_{A11}(-x^{11}) & G_{11BCD}(x^{11}) &= G_{11BCD}(-x^{11}) \\
g_{11,11}(x^{11}) &= g_{11,11}(-x^{11} & C_{ABC}(x^{11}) &= -C_{ABC}(-x^{11}) \\
& & C_{11BC}(x^{11}) &= C_{11BC}(-x^{11})
\end{aligned} \tag{12}$$

Clearly, there is no condition on the gauge fields, since they are defined on the Z_2 hyperplanes only. Furthermore, to check the number of preserved supersymmetries for the

solutions we are going to consider, we will need the supersymmetry transformations of the gravitino Ψ_M and the gauginos χ^α

$$\delta\Psi_M = D_M\eta + \frac{\sqrt{2}}{288}\left(\Gamma_M{}^{IJKL} - 8\delta_M^I\Gamma^{JKL}\right)\eta G_{IJKL} + \cdots \tag{13}$$

$$\delta\chi^\alpha = -\frac{1}{4}\Gamma^{AB}F_{AB}^\alpha\eta + \cdots . \tag{14}$$

The dots denote terms that involve the fermion fields of the theory. These terms vanish for the purely bosonic solution we are interested in. The 11–dimensional gamma matrices obey the condition $\{\Gamma_M, \Gamma_N\} = 2g_{MN}$. In order to keep the transformation (13) compatible with the Z_2 symmetry, the 11–dimensional Majorana spinor η has to be restricted by

$$\eta(x^{11}) = \Gamma_{11}\eta(-x^{11}) . \tag{15}$$

Note that this condition by itself does not restrict the number of 11–dimensional supersymmetries. On the Z_2 hyperplane, however, we have the chirality condition $\eta(0) = \Gamma_{11}\eta(0)$ which leads to the correct amount of supersymmetry, $N = 1$, in 10 dimensions.

3   The membrane

We are now ready to discuss BPS solutions of the above theory. To this end, it is useful to observe that the Yang–Mills boundary theory $\mathcal{L}_{\mathrm{YM}}$ in eq. (1), as well as the nontrivial term in the Bianchi identity (4), are suppressed by $\kappa^{2/3}$ with respect to the bulk theory $\mathcal{L}_{\mathrm{SG}}$. To lowest order in κ, the theory can thus be viewed as 11–dimensional supergravity subject to the Z_2 constraints (12). One approach to finding BPS solutions is, therefore, to start with such a solution of 11–dimensional supergravity; that is, with the elementary BPS membrane or the solitonic BPS 5–brane, and analyze to what extent it generalizes to a BPS solution of M–theory on S^1/Z_2. This requires a discussion of Z_2 invariance as well as $\kappa^{2/3}$ corrections. We will follow this approach for the $D = 11$ membrane and 5–brane.

Let us briefly review the multi–membrane solution of 11–dimensional supergravity [9, 7]. It is specified by the Ansatz [9, 7]

$$ds^2 = e^{2A}dx^\mu dx^\nu \eta_{\mu\nu} + e^{2B}dy^m dy^n \delta_{mn}$$

$$C_{\mu\nu\rho} = \pm\frac{1}{6\sqrt{2}\,^3g}\epsilon_{\mu\nu\rho}e^C \tag{16}$$

$$G_{m\mu\nu\rho} = \pm\frac{1}{\sqrt{2}\,^3g}\epsilon_{\mu\nu\rho}\partial_m e^C .$$

Here x^μ are the $2+1$ worldvolume coordinates labeled by indices $\mu, \nu, \ldots$ and y^m are the 8 transverse coordinates labeled by indices $m, n, \ldots$. Furthermore, ^3g is the determinant of

the worldvolume part of the metric. The functions A, B, C depend on the transverse coordinates y^m only. This Ansatz represents a multi–membrane solution of 11–dimensional supergravity (strictly, coupled to the 11–dimensional supermembrane action) provided that $A = C/3$ and $B = -C/6$. Here e^{-C} should be a harmonic function; that is, it should fulfill $\Box e^{-C} = 0$ away from the singularities where $\Box \equiv \delta^{mn} \partial_m \partial_n$ is the transverse Laplacian. For a solution corresponding to membrane sources at $\mathbf{y}^{(i)} = (y^{(i)m})$, the harmonic function e^{-C} can be written as

$$e^{-C} = 1 + \sum_i \frac{1}{|\mathbf{y} - \mathbf{y}^{(i)}|^6} \ . \tag{17}$$

With the above relations between A, B and C, the supersymmetry variation of the gravitino in eq. (13) vanishes for spinors η of the form

$$\eta = \epsilon \otimes \rho \ , \quad \rho = \rho_0 \, e^{C/6} \ , \tag{18}$$

with constant 3– and 8–dimensional spinors ϵ, ρ_0 and ρ_0 satisfying the chirality condition

$$(1 \pm \sigma)\rho_0 = 0 \ . \tag{19}$$

In these formulae, the 11–dimensional gamma matrices have been split as $\Gamma_M = \{\gamma_\mu \otimes \sigma, 1 \otimes \sigma_m\}$, where γ_μ, σ_m are the 3– and 8–dimensional gamma matrices, respectively, and $\sigma = \prod_m \sigma_m$. The projection condition (19) states that the solution preserves $1/2$ of the original $D = 11$ supersymmetry. The sign in eq. (19) which determines the chirality of the preserved supersymmetry is the same as in the Ansatz (16).

Next, we would like to embed this solution into M–theory on S^1/Z_2. This requires, as a first step, a discussion of its Z_2 properties. There are two different ways to orient the membranes with respect to the x^{11}–direction, namely to choose x^{11} as a worldvolume or a transverse coordinate. In the first case, the membranes stretch between the two Z_2 hyperplanes and intersect them as $1+1$–dimensional extended objects; that is, as strings. In the second case, the membranes are parallel to the Z_2 hyperplanes. Let us first assume that x^{11} is a worldvolume direction. In this case, there is no explicit dependence on x^{11} in the solution (16), so that the only nonvanishing fields should be the Z_2–even ones. A comparison with the Z_2 conditions (12) shows that indeed all fields in the eq. (16) are Z_2 even. The "orthogonal" membrane therefore automatically satisfies the orbifold constraint. In addition, to find the number of preserved supersymmetries, we have to implement the Z_2 constraint (15) on the spinor η. With $\Gamma_{11} = \gamma_{11} \otimes \sigma$, eqs. (15), (19) imply

$$(1 \pm \sigma)\rho_0 = 0 \ , \quad (1 \pm \gamma_{11})\epsilon = 0 \ , \tag{20}$$

for η as defined in eq. (18). Again, the sign which determines the chirality of the preserved supersymmetry is the same as in the Ansatz (16). These conditions show that the

membrane solution preserves 1/4 of the 11–dimensional supersymmetry of M–theory on S^1/Z_2 and 1/2 of the supersymmetry on the 10–dimensional hyperplanes. It is in this sense that we will use the term "BPS state of M–theory on S^1/Z_2" in the following. So far we have only considered terms to lowest order in κ. How is the solution affected by the corrections of order $\kappa^{2/3}$? Since we have not turned on gauge fields, the only source of such a correction is the $\text{tr}(R^2)$ term in the Bianchi identity (4). It is, however, straightforward to show that $\text{tr}(R^2)$ vanishes for the Ansatz (16) and, consequently, the solution does not receive any corrections of order $\kappa^{2/3}$. To summarize, we have therefore seen that the BPS membrane solution of $D = 11$ supergravity stretched between the Z_2 hyperplanes is also a BPS solution of M–theory on S^1/Z_2, including corrections of relative order $\kappa^{2/3}$. Upon restriction to the hyperplanes, the solution reduces to a string solution in the same way the membrane of $D = 11$ supergravity reduces to a string by dimensional reduction of one of its worldvolume coordinates [9]. This result extends immediately to multi–membrane solutions ending in multi–strings on the boundary hyperplanes.

It remains to show that the singularities in the above solution arise from supermembrane source terms [9]. For this to be the case, the source terms and, hence, the supermembrane equations of motion, must be compatible with the Z_2 orbifold symmetry. The gauge anomaly of the supermembrane worldvolume action embedded into a target space manifold with boundaries has been discussed in ref. [10]. Since, for the membrane solution, the gauge and gravitational anomalies are not switched on, we are allowed to consider the simple supermembrane action without anomaly cancellation terms for our purpose. The bosonic part of this action is

$$S_{\text{SM}} = \int d^3\xi \left(-\frac{1}{2}\sqrt{-\gamma}\gamma^{ij}\partial_i X^M \partial_j X^N g_{MN} + \frac{1}{2}\sqrt{-\gamma} \pm \frac{1}{3!}\epsilon^{ijk}\partial_i X^M \partial_j X^N \partial_k X^P C_{MNP} \right) \tag{21}$$

where ξ^i, $i = 0, 1, 2$ are the worldvolume coordinates, γ_{ij} is the worldvolume metric and X^M are the 11–dimensional target space coordinates. It follows from the identification of X^M with x^M in the equations of motion for $S + S_{\text{SM}}$ that the target space coordinates should transform as $X^A \to X^A$, $X^{11} \to -X^{11}$ under the Z_2 symmetry. Furthermore, this identification tells us that the background fields $g_{MN}(X^Q)$ and $C_{MNP}(X^Q)$ should satisfy the Z_2 conditions (12) with x^M replaced by X^M. Though the following discussion can be carried out in general, it is enough for our purpose to consider the specific gauge

$$X^\mu = \xi^\mu \quad \mu = 0, 1, 11 , \tag{22}$$

which is adapted to the orientation of the membrane worldvolume parallel to the orbifold direction. Clearly, this leads to the Z_2 transformation $\xi^i \to \xi^i$, $i = 0, 1$ and $\xi^{11} \to -\xi^{11}$ for the worldvolume coordinates. Then it is straightforward to show that the membrane equations of motion are Z_2 covariant provided we require for the worldvolume metric that

$$\gamma_{ij}(\xi^{11}) = \gamma_{ij}(-\xi^{11})$$

$$\gamma_{i11}(\xi^{11}) = -\gamma_{i11}(-\xi^{11})$$
$$\gamma_{11,11}(\xi^{11}) = \gamma_{11,11}(-\xi^{11}) \tag{23}$$

where $i, j = 0, 1$. This follows easily from the γ–equation of motion, $\gamma_{ij} = \partial_i X^M \partial_j X^N g_{MN}$, using the above gauge choice and the Z_2 properties of the metric. We conclude that, for an appropriate extension of the Z_2 symmetry to the worldvolume coordinates and a restriction of the worldvolume metric as above, the supermembrane equations of motion are Z_2 covariant. The explicit solution which we need to support the singularity is, in the $X^\mu = \xi^\mu$, $\mu = 0, 1, 11$ gauge, $X^m = \text{const}$, $m = 3, ..., 9$ and $\gamma_{\mu\nu} = e^{2A}\eta_{\mu\nu}$. According to the above rules, this solution indeed respects the Z_2 symmetry and, therefore, provides an acceptable source term for the "parallel" membrane solution of M–theory on S^1/Z_2. In addition, we should check that this solution does not break any of the preserved supersymmetries. This can be done in exactly the same way as for the ordinary 11–dimensional membrane [9] and is guaranteed by the condition (19).

Let us now address the case of x^{11} as a transverse direction. First, we should guarantee the Z_2 invariance of the harmonic function e^{-C}. This is easily done by pairing each membrane source at $y^{(i)11}$ with a "mirror source" at $-y^{(i)11}$ in the expression (17). Then all metric components in eq. (16) are Z_2 invariant. The components $G_{11\mu\nu\rho}$ of the 4–form in eq. (16), however, are proportional to $\partial_{11}e^C$ which changes sign under $x^{11} \to -x^{11}$. This is in conflict with the Z_2 conditions (12). One can cure this problem by using the additional sign freedom in eq. (16); that is, by choosing the $+$ sign for $x^{11} \in [0, \pi\rho]$ and the $-$ sign for $x^{11} \in [-\pi\rho, 0]$. Previously we had the chirality conditions (20) either for the positive or the negative sign. Now the conditions (20) have to be simultaneously fulfilled for both signs, so that all components of the spinor are projected out. Therefore, though this is a way of constructing a solution of M–theory on S^1/Z_2 based on the 11–dimensional membrane parallel to the hyperplanes, this solution does not respect any of the supersymmetries of M–theory on S^1/Z_2. Clearly, by the same argument as previously, the solution receives no corrections from $\kappa^{2/3}$ terms.

4   The 5–brane

Next, we will carry out a similar discussion for the 5–brane of $D = 11$ supergravity [11]. The Ansatz for this solution is given by

$$
\begin{aligned}
ds^2 &= e^{2A}dx^\mu dx^\nu \eta_{\mu\nu} + e^{2B}dy^m dy^n \delta_{mn} \\
G_{mnrs} &= \pm\frac{1}{\sqrt{2}}e^{-8B}\epsilon_{mnrs}{}^t\partial_t e^C ,
\end{aligned} \tag{24}
$$

where $\mu, \nu, ...$ label time and the 5 spatial worldvolume directions and $m, n, ...$ the 5 transverse directions. This Ansatz solves the equations of motion of 11–dimensional su-

pergravity provided $A = -C/6$, $B = C/3$ and $e^{-2C}\Box e^C = 0$, where $\Box = \delta^{mn}\partial_m\partial_n$. For 5–branes at $\mathbf{y}^{(i)} = (y^{(i)m})$ the harmonic function e^C can be written in the form

$$e^C = 1 + \sum_i \frac{1}{|\mathbf{y} - \mathbf{y}^{(i)}|^3} \ . \tag{25}$$

With the above relations between A, B and C, the gravitino supersymmetry variation (13) vanishes for spinors

$$\eta = \epsilon \otimes \rho \ , \quad \rho = e^{-C/12}\rho_0 \tag{26}$$

with

$$(1 \pm \gamma)\epsilon = 0 \ . \tag{27}$$

Here, ϵ, ρ_0 are constant 6– and 5–dimensional spinors, respectively. The sign in eq. (27) which determines the chirality of the unbroken supersymmetries is the same as the one in the Ansatz (24). We have decomposed the $D = 11$ gamma matrices as $\Gamma_m = \{\gamma_\mu \otimes 1, \gamma \otimes \sigma_m\}$ with $\gamma = \prod_\mu \gamma_\mu$. As before, to discuss Z_2 invariance, we distinguish the two cases of x^{11} being a worldvolume or a transverse direction. Let us start with the latter case. Z_2 invariance of the harmonic function e^C is achieved by pairing each 5–brane at $y^{(i)11}$ with a mirror 5–brane at $-y^{(i)11}$ in eq. (25). Comparison with the Z_2 condition (12) shows that this guarantees a Z_2–even solution. Using $\Gamma_{11} = \gamma \otimes \sigma_{11}$ and the eqs. (15), (27) we get

$$(1 \pm \gamma)\epsilon = 0 \ , \quad (1 \pm \sigma_{11})\rho_0 = 0 \ . \tag{28}$$

Therefore $1/4$ of the 11-dimensional supersymmetry and $1/2$ of the 10–dimensional supersymmetry of M–theory on S^1/Z_2 is preserved. As in the membrane, $\text{tr}(R^2)$ vanishes for the metric (24) so that the "parallel" 5–brane receives no corrections of order $\kappa^{2/3}$. In summary, we have seen that the 11–dimensional 5–brane oriented parallel to the Z_2 hyperplanes is a BPS solution (understood in the sense explained above) of M–theory on S^1/Z_2 including terms of order $\kappa^{2/3}$.

For the 5–brane stretched between the hyperplanes, we face a similar situation as for the "parallel" membrane. In this case, the function e^C is automatically Z_2 invariant. The components G_{mnrs} in eq. (24), however, do not change sign at the hyperplanes as required by Z_2 invariance unless we use the sign freedom in the Ansatz (24). As before, this means that we choose the $+$ sign for $x^{11} \in [0, \pi\rho]$ and the $-$ sign for $x^{11} \in [-\pi\rho, 0]$. This leads to a solution of M–theory on S^1/Z_2 including $\kappa^{2/3}$ terms. However, since the sign in the Ansatz (24) is linked to the sign in the chirality condition (27), eq. (28) holds for both signs simultaneously. Consequently, no supersymmetries are preserved.

5 The gauge 5–brane

The solutions derived from 11–dimensional supergravity which we have considered so far did not lead to any nontrivial term on the right hand side of the Bianchi identity (4),

since $\text{tr}(R^2) = 0$. A way to obtain a nontrivial Bianchi identity, is to consider a solution with a nonvanishing gauge field configuration, so that $\text{tr}(F^2) \neq 0$. For the weakly coupled heterotic string, such a solution is given by the gauge 5–brane of ref. [6]. We are now going to analyze the analog of this solution in the strongly coupled case.

Since $\text{tr}(R^2) = 0$ for the metric which we are going to consider, the Bianchi identity (4) now reads

$$(dG)_{11ABCD} = -3\sqrt{2}\frac{\kappa^2}{\lambda^2}\left(\delta(y^{11})\text{tr}(F^{(1)}_{[AB}F^{(1)}_{CD]}) + \delta(y^{11} - \pi\rho)\text{tr}(F^{(2)}_{[AB}F^{(2)}_{CD]})\right) , \qquad (29)$$

where we have included gauge fields strengths $F^{(1)}$ and $F^{(2)}$ for both hyperplanes. Later on, we will find it useful to solve this Bianchi identity, as well as the equation of motion for G, in the boundary picture [2] as opposed to the orbifold picture which we have used so far. In this picture, we think of the 11–dimensional space as the interval $0 \leq x^{11} \leq \pi\rho$ times a 10–dimensional manifold. Then the source terms in the orbifold picture turn into boundary conditions at the two boundaries $x^{11} = 0, \pi\rho$ of the 11–dimensional manifold. More explicitly, one can determine these boundary conditions by solving the above Bianchi identity close to the fixed points [2]. The resulting conditions are

$$G_{ABCD}|_{x^{11}=0} = -\frac{3}{\sqrt{2}}\frac{\kappa^2}{\lambda^2}\text{tr}(F^{(1)}_{[AB}F^{(1)}_{CD]})$$

$$G_{ABCD}|_{x^{11}=\pi\rho} = +\frac{3}{\sqrt{2}}\frac{\kappa^2}{\lambda^2}\text{tr}(F^{(2)}_{[AB}F^{(2)}_{CD]}) . \qquad (30)$$

One then solves the homogeneous Bianchi identity $dG = 0$ instead of eq. (29), with the equation of motion (10) for G being subject to the conditions (30).

Let us now set up the Ansatz for the gauge 5–brane solutions. The previous experience with the ordinary 5–brane leads us to orient the x^{11}–direction in the transverse space in order to preserve some supersymmetries. We therefore start with the Ansatz

$$ds^2 = e^{2A}dx^\mu dx^\nu \eta_{\mu\nu} + e^{2B}dy^m dy^n \delta_{mn} \qquad (31)$$

for the metric, where $\mu, \nu, \ldots = 0, \ldots, 5$ label the worldvolume directions and $m, n, \ldots = 6, \ldots, 9, 11$ the transverse directions including x^{11}. We also introduce indices $a, b, \ldots = 6, \ldots, 9$ for the transverse directions orthogonal to the orbifold. For the 4–form we write, in analogy with the ordinary 5–brane,

$$G_{mnrs} = \pm\frac{1}{\sqrt{2}}e^{-8B}\epsilon_{mnrs}{}^t\partial_t e^C . \qquad (32)$$

Finally, we have to specify the gauge fields. We consider simple $SU(2)$ instantons [12] on both Z_2 hyperplanes specified by $A^{(1,2)i}_a = \eta^{(1,2)i}_{ab}\partial_b h_{1,2}$ where $h_{1,2} = -\ln f_{1,2}$. Here

the tensors $\eta^{(1,2)i}_{ab}$ are defined by $\eta^{(1,2)i}_{ab} = \epsilon_{0iab} + \epsilon^{(1,2)}(\delta_{ia}\delta_{b0} - \delta_{ib}\delta_{a0})$ and the function f satisfies $f^{-1}\Box_4 f = 0$ with the 10–dimensional transverse Laplacian $\Box_4 \equiv \delta^{ab}\partial_a\partial_b$. The signs $\epsilon^{(1,2)} = \pm$ in the definition of $\eta^{(1,2)i}_{ab}$ specify whether the instanton is selfdual ($+$ sign) or anti–selfdual ($-$ sign). The $SU(2)$ generators are chosen as $T^i = \tau^i/2i$ with the Pauli matrices τ^i. For the gauge fields defined in such a way, one finds $\mathrm{tr}(F^{(1,2)}_{[ab}F^{(1,2)}_{cd]}) = \pm\frac{1}{6}\hat{\epsilon}_{abcd}\Box^2_4 h_{1,2}$ where $\hat{\epsilon}_{abcd}$ is the flat totally antisymmetric ϵ tensor. Thus, the right hand side of the Bianchi identity (29) or, equivalently, the boundary conditions (30) are completely determined.

We are now ready to discuss the equations of motion. Since the Ansätze for the metric and the 4–form are identical to the ones for the ordinary 5–brane, it is natural to look for a solution which fulfills the familiar relations $A = -C/6$, $B = C/3$. Indeed, using these relations one finds that the (mn) components of the Einstein equation (8), the equation of motion (10) for G and the equations of motion for the gauge fields (11) are identically fulfilled. The remaining equations are the $(\mu\nu)$ components of the Einstein equation and the Bianchi identity (29) which both contain gauge field source terms. If we choose the instantons on the hyperplanes to be of the same type (both selfdual or both anti–selfdual) and, in addition, choose (anti)–selfdual instantons for the $+$ $(-)$ sign in the Ansatz (32) for G ($\epsilon^{(1)} = \epsilon^{(2)} = \pm$), then these two equations are, in fact, identical to

$$\Box_4 e^C = \delta(y^{11})J_1 + \delta(y^{11} - \pi\rho)J_2 \ . \tag{33}$$

Here $\Box_5 \equiv \delta^{mn}\partial_m\partial_n$ and J_1, J_2 are the instanton source terms explicitly given by

$$J_{1,2} = -\frac{\kappa^2}{2\lambda^2}\Box^2_4 h_{1,2} \tag{34}$$

with the functions $h_{1,2}$ as defined above. The remaining problem is to solve equation (33), and we will find it useful to do this in the boundary picture. Following the steps explained in the beginning of this section, the equivalent problem in the boundary picture can be formulated as

$$\Box_5 \Phi(x^a; x^{11}) = 0 \tag{35}$$

with boundary conditions

$$\partial_{11}\Phi|_{x^{11}=0} = \frac{J_1(x^a)}{2}, \ \partial_{11}\Phi|_{x^{11}=\pi\rho} = -\frac{J_2(x^a)}{2} \tag{36}$$

Here we have defined $\Phi = e^C$. In this formulation, it is obvious that the solution will have a nontrivial dependence on the x^{11}–coordinate since the field Φ has to interpolate between the "surface charges" on the boundaries provided by the instantons. More specifically, since M–theory on S^1/Z_2 turns into weakly coupled heterotic string theory upon shrinking the x^{11} direction, we expect an x^{11}–independent bulk component of Φ which, in some sense (to be precisely specified later), corresponds to the weakly coupled gauge 5–brane. On top

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of this bulk component, Φ contains an x^{11}–dependent piece which represents the strong coupling regime "dressing" of the gauge 5–brane. In general, this solution for Φ cannot be expressed in terms of the instanton sources $J_{1,2}$ in a simple way. Noticing that eqs. (35), (36) constitute a problem in potential theory with von Neumann boundary conditions, one might apply methods familiar from classical electrodynamics to find a solution. Here, we will, however, use a more direct approach which is better suited to the expected structure of the solution. Let us split Φ into two parts as

$$\Phi = \Phi_0 + \phi, \tag{37}$$

where ϕ is a function of x^m, $m = 6, 7, 8, 9, 11$, and Φ_0 is a function of x^a, $a = 6, 7, 8, 9$ only. The average $< ... >$ over the 11-th dimension is defined as $< f > = \frac{1}{\pi\rho} \int_0^{\pi\rho} f(x^{11})dx^{11}$. We further demand that

$$< \phi > = 0. \tag{38}$$

The condition $< \phi > = 0$ can always be achieved by a redefinition of Φ_0, and it determines the decomposition (37) uniquely. The idea is that the x^{11}–independent piece Φ_0 is the zero mode of Φ and will eventually correspond to the 10–dimensional gauge 5–brane, whereas ϕ represents the x^{11}–dependent corrections. Inserting this Ansatz into eq. (35) one gets

$$\Box_5\Phi = \Box_4\Phi_0 + \Box_4\phi + \partial_{11}^2\phi = 0. \tag{39}$$

In addition, it follows from eq. (36) that ϕ has to fulfill the boundary condition

$$\partial_{11}\phi|_{x^{11}=0} = \frac{J_1(x^a)}{2}, \quad \partial_{11}\phi|_{x^{11}=\pi\rho} = -\frac{J_2(x^a)}{2}. \tag{40}$$

By integrating over x^{11}, taking into account $< \phi > = 0$ and eq. (40), we arrive at a purely 4–dimensional equation for Φ_0

$$\Box_4\Phi_0 = \frac{1}{2\pi\rho}(J_1 + J_2). \tag{41}$$

On the other hand, using this result to eliminate Φ_0 from eq. (39) we find ϕ to be determined by

$$\Box_4\phi + \partial_{11}^2\phi = -\frac{1}{2\pi\rho}(J_1 + J_2). \tag{42}$$

What we have achieved so far is to split the original equation into an x^{11}–independent equation (41) for Φ_0, and a modified boundary value problem for ϕ. Together, the equations (41), (42) and (40) are completely equivalent to the original problem. If we put $J_2 = 0$, eq. (41) is just the same equation that arises for the weakly coupled gauge 5–brane. Its solutions are therefore well known [6]. Consider the case of two instantons, one located on the $x^{11} = 0$ hyperplane at $r = 0$ (where $r \equiv \sqrt{y^a y^b \delta_{ab}}$ is the 4–dimensional

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radius) with size specified by σ_1 and $h_1 = -\ln(1 + \frac{\sigma_1^2}{r^2})$, and the other on the $x^{11} = \pi\rho$ hyperplane, also at $r = 0$, with size σ_2 and $h_2 = -\ln(1 + \frac{\sigma_2^2}{r^2})$. Then the nonsingular solution of equation (41) is given by

$$\Phi_0 = 1 + \frac{2\kappa^2}{\pi\rho\lambda^2}\left(\frac{2\sigma_1^2 + r^2}{(\sigma_1^2 + r^2)^2} + \frac{2\sigma_2^2 + r^2}{(\sigma_2^2 + r^2)^2}\right) . \tag{43}$$

The arbitrary additive constant in Φ_0 has been normalized to 1, so that the physical distance between the two hyperplanes far away from the instanton core ($r \to \infty$) equals the coordinate distance $\pi\rho$. Note that allowing formally $\sigma_2 \to 0$, and subtracting $\frac{2\kappa^2}{\pi\rho\lambda^2 r^2}$ which is a solution to the homogeneous equation, corresponds to the special case of an instanton at $x^{11} = 0$ but no instanton on the $x^{11} = \pi\rho$ hyperplane. In this case, the above solution becomes

$$\Phi_0 = 1 + \frac{2\kappa^2}{\pi\rho\lambda^2}\frac{2\sigma^2 + r^2}{(\sigma^2 + r^2)^2} . \tag{44}$$

which corresponds to Strominger's original solution. The generalization of this solution to two instantons located at different positions, as well as to the multi–instanton case, is straightforward.

The final task is to find the solution for ϕ which contains the nontrivial dependence on x^{11}. In general, the solution of eq. (42) is very complicated because of the mixing of x^{11} with the 4–dimensional coordinates. A solution can, however, be obtained by considering the following expansion

$$\phi = \sum_{n=0}^{n=\infty} \psi_n \tag{45}$$

where

$$\psi_n = P_n(x^{11})\Box_4^n J_1 + Q_n(x^{11})\Box_4^n J_2 . \tag{46}$$

Here, P_n and Q_n are functions of x^{11}. Inserting this series into eq. (42), and applying the boundary conditions (40) as well as $< \phi >= 0$, we arrive at the recurrent series of equations

$$\partial_{11}^2\psi_0 = -\frac{1}{2\pi\rho}(J_1 + J_2), \quad \partial_{11}\psi_0|_{x^{11}=0} = \tfrac{1}{2}J_1, \quad \partial_{11}\psi_0|_{x^{11}=\pi\rho} = -\frac{1}{2}J_2$$

$$\partial_{11}^2\psi_n = -\Box_4\psi_{n-1}, \quad \partial_{11}\psi_n|_{x^{11}=0,\pi\rho} = 0, \quad n = 1, 2, ... \tag{47}$$

and

$$< \psi_n >= 0, \quad n = 0, 1, 2, ... \tag{48}$$

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This series has a recursive solution in terms of polynomials P_n, Q_n, where

$$P_0 = -\frac{(x^{11})^2}{4\pi\rho} + \frac{x^{11}}{2} - \frac{\pi\rho}{6} , \quad Q_0 = -\frac{(x^{11})^2}{4\pi\rho} + \frac{\pi\rho}{12} \tag{49}$$

and

$$\partial_{11}^2 P_n = -P_{n-1} , \qquad \partial_{11}^2 Q_n = -Q_{n-1}$$
$$\partial_{11} P_n|_{x^{11}=0} = 0 , \qquad \partial_{11} Q_n|_{x^{11}=0} = 0 \tag{50}$$
$$< P_n >= 0 , \qquad < Q_n >= 0 , \tag{51}$$

for $n > 0$. For consistency, we have to check that the polynomials P_n, Q_n can really be chosen to fulfill all three conditions; that is, the two boundary conditions and the vanishing average condition. This is, à priori, not obvious since they are obtained by integrating a second order differential equation and, therefore, contain only two free parameters. Luckily, using the differential equation for P_n, the vanishing of the average, $< P_{n-1} >= 0$ implies that P_n automatically satisfies the correct boundary condition at $x^{11} = \pi\rho$ (and the same for Q_n). The other two conditions can be fulfilled by adjusting the two integration constants so that P_n is uniquely determined. It is easy to compute the successive polynomials. For example,

$$P_1 = \frac{(x^{11})^4}{48\pi\rho} - \frac{(x^{11})^3}{12} + \frac{\pi\rho(x^{11})^2}{12} - \frac{\pi^3\rho^3}{90}, \quad Q_1 = \frac{(x^{11})^4}{48\pi\rho} - \frac{\pi\rho(x^{11})^2}{24} + \frac{7\pi^3\rho^3}{720} \tag{52}$$

Now, the question arises as to how quickly the series solution constructed above converges. As one can easily check, the ratio of two successive terms in the series (45) is given by $\psi_n/\psi_{n-1} = O\left((\pi\rho)^2/\sigma^2\right)$, where σ is the scale over which the gauge field varies (the instanton size). Formally, our series provides a solution for any value of this ratio. For very small instantons, however, the series might converge poorly. If, on the other hand, the instanton size is sufficiently large as compared to the separation $\pi\rho$ of the boundaries, the series rapidly converges and the solution for ϕ is well approximated by the first few terms in the series. Let us again consider the case of two instantons, one of size σ_1 located at $r = 0$ on the $x^{11} = 0$ hyperplane, and the other of size σ_2 located at $r = 0$ at $x^{11} = \pi\rho$. If we assume that $\sigma_i \gg \pi\rho$ for $i = 1, 2$, then the solution for ϕ is well approximated by the first two terms in the series (46). It follows from (34),(46),(49),(52) that $\phi = \psi_0 + \psi_1 + \cdots$ where

$$\psi_0 = -48\frac{\kappa^2}{\lambda^2}\left(P_0(x^{11})\frac{\sigma_1^4}{(\sigma_1^2 + r^2)^4} + Q_0(x^{11})\frac{\sigma_2^4}{(\sigma_2^2 + r^2)^4}\right) \tag{53}$$

and

$$\psi_1 = -768\frac{\kappa^2}{\lambda^2}\left(P_1(x^{11})\frac{3r^2 - 2\sigma_1^2}{(\sigma_1^2 + r^2)^6} + Q_1(x^{11})\frac{3r^2 - 2\sigma_2^2}{(\sigma_2^2 + r^2)^6}\right) . \tag{54}$$

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We emphasize that this x^{11}-dependent solution represents a true strong coupling correction to the gauge 5-brane.

As an example, in Figure 1, we have plotted ϕ as a function of $x = x^{11}$ and the 4–dimensional radius r interpolating between two slightly different instantons located opposite to each other on the two boundaries. The separation of the boundaries has been chosen as $\rho = 1$ and the instantons at $x^{11} = 0$, $r = 0$ and $x^{11} = \pi$, $r = 0$ have the size $\sigma_1 = 11$ and $\sigma_2 = 10$, respectively. It was sufficient to use the first two terms in the series (45) only.
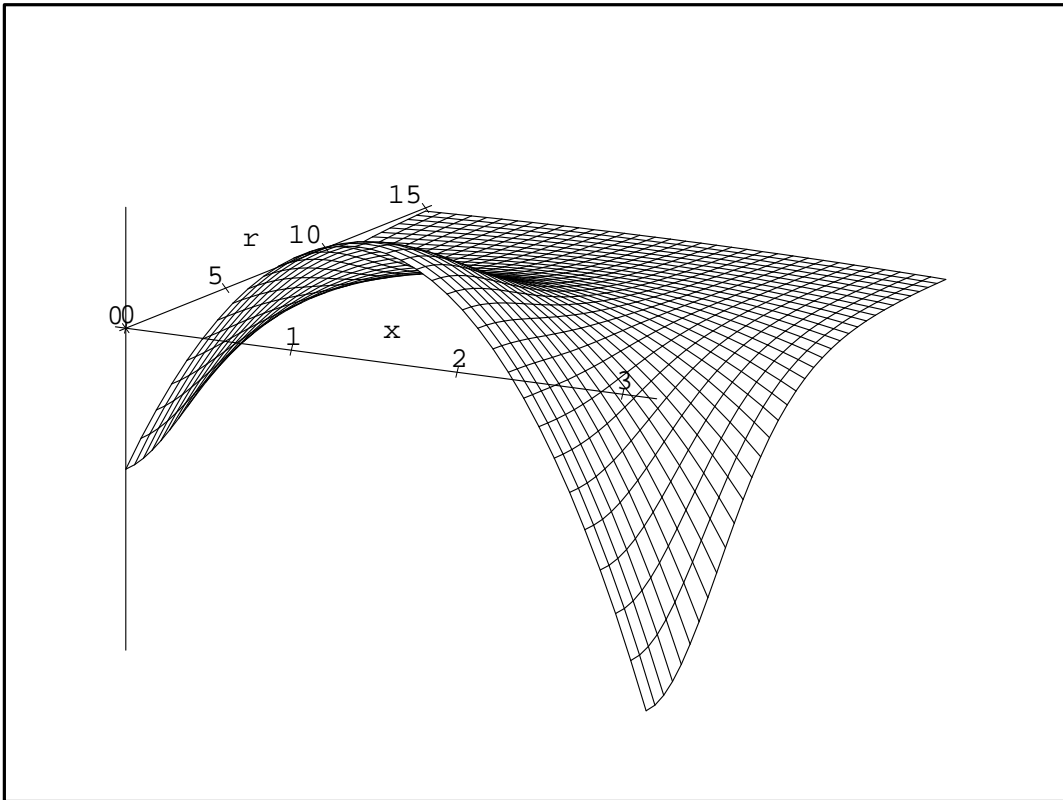

Fig 1: Correction ϕ to the string coupling interpolating between two instantons.

We have not yet checked whether our solution preserves any supersymmetries. This is, however, easily done since the vanishing of the supersymmetry variation of the gravitino does not depend on the explicit solution for $e^C = \Phi$, but rather on the structure of the Ansatz and the relations between A, B, C. Both are identical to the ones for the ordinary 5–brane. Consequently, for spinors η of the form

$$\eta = \epsilon \otimes \rho \,, \quad \rho = \rho_0 \, e^{A/2} \tag{55}$$

and

$$(1 \pm \gamma)\epsilon = 0 \tag{56}$$

the gravitino supersymmetry variation (13) vanishes [1]. Here, the chirality sign is the same as the one in the Ansatz (32) for G, and is consequently $+$ $(-)$ for (anti)–selfdual instantons. Recall that, in the construction of the solution, we have chosen the instantons on both boundaries to be of the same type. Here we find that this is necessary to preserve any supersymmetries. Along with the Z_2 condition $(1 \pm \sigma_{11})\rho_0 = 0$, eq. (56) implies that the solution preserves $1/4$ of the 11-dimensional supersymmetry and $1/2$ of the 10–dimensional supersymmetry of M–theory on S^1/Z_2. If we had chosen instantons of different types, we would find two chirality conditions of opposite sign from the two boundaries, thereby projecting out the full spinor.

Finally, we would like to discuss the precise relation of our 11–dimensional solution to the corresponding 10–dimensional gauge 5–brane. Generally, since our solution is the strongly coupled version of the gauge 5–brane, we expect it to consist of a x^{11}-independent bulk piece identical to the weakly coupled gauge 5–brane plus x^{11}–dependent strong coupling corrections. The field strength H_{abc} of the 10–dimensional Neveu–Schwarz 2–form is the zero mode of G_{abc11}, which can be computed by averaging

$$H_{abc} \equiv \frac{1}{\sqrt{2}} < G_{abc11} > = \mp \frac{1}{2} \hat{\epsilon}_{abc}{}^d \partial_d < e^C > \ . \tag{57}$$

Let φ be the 10–dimensional dilaton related to the string coupling g_S by $g_S = e^{-\varphi}$. Then, a comparison of the above equation with the Ansatz for the weakly coupled gauge 5–brane leads us to identify $g_S^2 = e^{-2\varphi} = < e^C > = \Phi_0$. Recall that Φ_0 is the x^{11}–independent part of our solution defined in eq. (37). With this identification, the Bianchi identity (41) for Φ_0 turns exactly into its 10–dimensional counterpart [6]. More generally, a reduction of the 11–dimensional action to 10 dimensions on a metric with $g_{11,11} = e^{2B}$ leads to the identification [13] $g_S^2 = e^{-2\varphi} = e^{3B}$. For our solution $B = C/3$ so that $g_S^2 = e^C$. Since the function C depends on x^{11}, this last equation is not quite correct, but should be replaced by its averaged version $g_S^2 = e^{-2\varphi} = < e^C > = \Phi_0$. This is in agreement with the previous result obtained by matching the form fields. The relation between the 11–dimensional metric g_{AB} and the 10–dimensional string frame metric ${}^{10}g_{AB}$ is given by ${}^{10}g_{AB} = e^{-2\varphi/3} g_{AB}$. Our solution (31) for the metric written in the 10–dimensional string frame then turns into

$$ds_{10}^2 = (1 + e^{2\varphi}\phi)^{-1/3} dx^\mu dx^\nu \eta_{\mu\nu} + e^{-2\varphi}(1 + e^{2\varphi}\phi)^{2/3} dy^a dy^b \delta_{ab} \ . \tag{58}$$

Upon dropping the higher Fourier modes modes of ϕ, this metric coincides with the one for the weakly coupled gauge 5–brane [6]. In conclusion, we have seen that splitting up

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[1]The decomposition for spinors and gamma matrices is the same as for the ordinary 5–brane explained below eq. (27).

our solution as $e^C = \Phi_0 + \phi$, with an x^{11}–independent piece Φ_0 and an x^{11}–dependent correction ϕ with $< \phi >= 0$, provides the correct correspondence to the 10–dimensional gauge 5–brane. In particular Φ_0 equals the square of the string coupling.

6   Conclusions

In this paper, we have considered soliton solutions of M–theory on S^1/Z_2, the low energy theory for the strongly coupled heterotic string. We have found that the membrane solution of $D = 11$ supergravity continues to be a solution of M–theory on S^1/Z_2, including terms of relative order $\kappa^{2/3}$ for membranes oriented orthogonal, as well as parallel, to the Z_2 hyperplanes. Only in the first case, however, does the solution constitute a BPS state of M–theory on S^1/Z_2. The term "BPS", in the present context, is understood to label solutions which preserve $1/4$ of the 11–dimensional supersymmetry and $1/2$ of the 10–dimensional supersymmetry (on the Z_2 hyperplanes) of M–theory on S^1/Z_2. Membranes oriented parallel to the hyperplanes, on the other hand, do not preserve any supersymmetries. This reflects the fact that an orthogonal BPS membrane leads, by dimensional reduction in the x^{11}–direction, to a BPS string, as desired for the weakly coupled heterotic theory. A parallel BPS membrane, on the other hand, would lead to a membrane of the weakly coupled heterotic theory, which does not exist. It is interesting to trace the nature of supersymmetry breaking for parallel membranes. The supersymmetry variation of the gravitino can be set to zero near any bulk point $x^{11} \in [-\pi\rho, \pi\rho]$ for appropriate spinors η. Globally, however, we are faced with the sign flip in the chirality condition (20) which projects out all globally defined spinors. Hence, all supersymmetries are broken. This mechanism is reminiscent of global supersymmetry breaking by gaugino condensation, discussed in ref. [14].

The situation for the 11–dimensional 5–brane is similar, but with the rôle of the orientations being reversed. For both possible orientations it is a solution of M–theory on S^1/Z_2. However, the solution is BPS for parallel 5–branes only. The orthogonal 5–brane, on the other hand, does not preserve any supersymmetry. Again, this reflects the properties of the weakly coupled heterotic theory in $D = 10$, which has a 5–brane but not a 4–brane solution.

The 10-dimensional gauge 5–brane generalizes to the full 11–dimensional theory in a nontrivial way. Unlike the membrane and 5-brane solutions, the gauge 5-brane does receive nontrivial corrections of order $\kappa^{2/3}$. This happens because the instantons on the hyperplanes switch on the anomalous terms in the Bianchi identity. We have presented a solution for this strongly coupled gauge 5-brane which makes its relation to the weakly coupled counterpart transparent. In particular, our solution contains an x^{11}-independent bulk component which in a case of a single instanton exactly coincides with the weakly coupled gauge 5-brane. On top of this component comes an x^{11}-dependent part which is

needed to interpolate between the instantons on different planes. It represents the strong coupling effect in the gauge 5-brane solution. This x^{11}-dependent part has been computed in an expansion scheme which is quickly converging as long as the gauge field varies slowly compared to the separation of the hyperplanes. However, it should be noted that formally the solution we have given solves the equation for ϕ in general. In the present case it is possible to approximate the solution very well just with the first two terms, but in more general cases the convergence may be much slower, in particular if one moves away from the hyperplanes. Clearly, this method of finding solutions to the Horava-Witten model is not restricted to instanton type configurations, but can be applied to any physical gauge configuration on the boundaries. It is also particularly well suited to analyze the relation of the 11-dimensional theory to its 10-dimensional limit. An explicit example of the x^{11}-dependent part of our solution for two instantons of different sizes located opposite to each other on different hyperplanes has been depicted in Figure 1. For instantons of the same type (both selfdual or both anti-selfdual) our solution preserves one-quarter of the 11-dimensional supersymmetry and one-half of the 10-dimensional supersymmetry.

These gauge 5-brane solutions are the first explicit examples of soliton solutions in the field-theoretical limit of M-theory on S^1/Z_2 which receive nontrivial strong-coupling corrections.

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